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LETTER TO THE EDITOR

On secondary and higher-generation ghosts

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Abstract. We give a simplified discussion of 'secondary' and further Faddeev–Popov ghost fields occurring in covariantly quantised gauge theories with differentially constrained gauge transformations. Several examples of this new phenomenon are presented.

In connection with recent work on the superfield quantisation of supergravity, it was discovered that in certain circumstances covariant quantisation leads to the occurrence of 'secondary' Faddeev–Popov ghost fields (Namazie and Storey 1979). These arise in gauge theories for which the gauge parameter is subject to a differential constraint (or, equivalently, when only a non-local projection of the parameter is actually involved in the gauge transformation). However, this rather simple but interesting feature was perhaps somewhat obscured in Namazie and Storey (1979) by the intricate supergravity formalism involved.

More recently, another example of this phenomenon has been provided by Townsend's (1979) discussion of the covariant quantisation of antisymmetric rank-two tensor gauge fields.

It is the purpose of this Letter to point out that these are particular examples of the more general phenomenon referred to above, and to give further examples of current interest.

To illustrate our remarks, we begin by briefly recapitulating the essential details of the antisymmetric rank-two tensor case (Townsend 1979), which is perhaps the simplest. The Lagrangian for the field $A_{\mu\nu}(=A_{[\mu\nu]})$ is given by

$$\mathscr{L}=\tfrac{3}{2}(\partial_{[\lambda}A_{\mu\nu]})^2$$

and is invariant under the gauge transformation

$$A_{\mu\nu}\rightarrow A_{\mu\nu}+\partial_{[\mu}\Lambda_{\nu]}.$$

Clearly only the transverse projection of Λ_ν is involved here, and if desired the differential condition

$$\partial_\mu\Lambda_\mu=0$$

could be stipulated. A suitable gauge-fixing constraint is

$$K_\mu\equiv 2\,\partial_\nu A_{\nu\mu}=0,$$

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and, to maintain manifest gauge invariance of the functional integral, the ghost Lagrangian

$$\mathscr{L}_G = B_\mu^{(1)}(\delta K_\mu/\delta\Lambda_\nu)B_\nu^{(2)} = B_\mu^{(1)}(\Box\eta_{\mu\nu} - \partial_\mu\,\partial_\nu)B_\nu^{(2)}$$

must be added, where $B_\mu^{(1)}$ and $B_\nu^{(2)}$ are the anticommuting (Feynman–DeWitt–) Faddeev–Popov ghost fields. We now observe that $\mathscr{L}_G$ itself has two gauge invariances, namely

$$B_\mu^{(1)} \to B_\mu^{(1)} + \partial_\mu\Sigma^{(1)} \qquad \text{and} \qquad B_\mu^{(2)} \to B_\mu^{(2)} + \partial_\mu\Sigma^{(2)},$$

where $\Sigma^{(1)}$ and $\Sigma^{(2)}$ are (independent) anticommuting parameters. Thus further gauge-fixing constraints are necessary and may be taken to be

$$\partial_\mu B_\mu^{(1)} = \partial_\mu B_\mu^{(2)} = 0.$$

These lead to the secondary ghost Lagrangian

$$\mathscr{L}'_G = C^{(1)}\Box C^{(2)} + C^{(3)}\Box C^{(4)},$$

where $C^{(1)}, \ldots, C^{(4)}$ are *commuting* scalars. The question of unitarity and the actual book-keeping of physical modes are discussed in detail in Townsend (1979). The number of surviving degrees of freedom is arrived at as follows. The effective Lagrangian (turning gauge-fixing constraints into gauge-fixing terms in the standard manner) is

$$\mathscr{L}_{\text{eff}} = \tfrac{3}{2}(\partial_{[\lambda}A_{\mu\nu]})^2 + (1/\alpha_A)(\partial_\nu A_{\nu\mu})^2 + B_\mu^{(1)}(\Box\eta_{\mu\nu} - \partial_\mu\partial_\nu)B_\nu^{(2)}$$
$$+ (1/\alpha_B)B_\mu^{(1)}\,\partial_\mu\,\partial_\nu B_\nu^{(2)} + C^{(1)}\Box C^{(2)} + C^{(3)}\Box C^{(4)}.$$

The $A_{\mu\nu}$ part of $\mathscr{L}_{\text{eff}}$ describes six modes; the B_μ part subtracts eight (since $B_\mu^{(1)}$ and $B_\nu^{(2)}$ are anticommuting); finally, the C terms add four, leaving a total of two degrees of freedom. One of these decouples due to Ward identities, and thus one is left with a single physical degree of freedom describing a scalar field.

A similar analysis may be applied to a totally antisymmetric rank-three tensor field, with amusing results. The Lagrangian is

$$\mathscr{L} = -2(\partial_{[\sigma}A_{\mu\nu\lambda]})^2 \qquad (1)$$

invariant under

$$A_{\mu\nu\lambda} \to A_{\mu\nu\lambda} + \partial_{[\mu}\Lambda_{\nu\lambda]}. \qquad (2)$$

Note that not all of the antisymmetric parameter $\Lambda_{\mu\nu}$ actually takes part in (2), but only the projection $P_{\mu\nu\sigma\tau}\Lambda_{\sigma\tau}$, where

$$P_{\mu\nu\sigma\tau} \equiv \tfrac{1}{2}(\eta_{\mu\sigma}\eta_{\nu\tau} - \eta_{\mu\tau}\eta_{\nu\sigma}) - (1/2\Box)(\eta_{\mu\sigma}\,\partial_\nu\,\partial_\tau + \eta_{\nu\tau}\,\partial_\mu\,\partial_\sigma - \eta_{\mu\tau}\,\partial_\nu\,\partial_\sigma - \eta_{\nu\sigma}\,\partial_\mu\,\partial_\tau),$$

and so one could impose the differential constraint

$$\partial_\mu\Lambda_{\mu\nu} = 0$$

to remove this redundancy.

This field is non-propagating in 3 + 1 dimensions, but has nevertheless found several applications of late. One such is in a four-dimensional extension of the Schwinger model, conjectured to be relevant to confinement (Aurilia 1979). In addition, the

Lagrangian (1) is identical to that of certain auxiliary fields occurring in supergravity, after a duality transformation; defining

$$A^*_\sigma \equiv (1/3!)\epsilon_{\sigma\mu\nu\lambda}A_{\mu\nu\lambda},$$

the Lagrangian (1) may be rewritten as

$$\mathscr{L} = 3(\partial_\sigma A^*_\sigma)^2$$

(cf Namazie and Storey (1979) for instance). Furthermore, such a field also appears in $10+1$-dimensional supergravity, which becomes $N=8$ extended supergravity after dimensional reduction (Cremmer and Julia 1979). (In $10+1$ dimensions it is in fact a propagating field.)

Applying the previous argument, one obtains the effective Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{eff}} = -2(\partial_{[\sigma}A_{\mu\nu\lambda]})^2 + (1/\alpha_A)(\partial_\mu A_{\mu\nu\lambda})^2 + B^{(1)}_{[\nu\lambda}\,\partial_{\mu]}\,\partial_{[\mu}B^{(2)}_{\nu\lambda]} \\ + (1/\alpha_B)(\partial_\mu B^{(1)}_{\mu\nu})(\partial_\lambda B^{(2)}_{\lambda\nu}) + C^{(1)}_\mu(\Box\eta_{\mu\nu} - \partial_\mu\,\partial_\nu)C^{(2)}_\nu \\ + C^{(3)}_\mu(\Box\eta_{\mu\nu} - \partial_\mu\,\partial_\nu)C^{(4)}_\nu + (1/\alpha_C)\,\partial_\mu C^{(1)}_\mu\,\partial_\nu C^{(2)}_\nu + (1/\alpha'_C)\,\partial_\mu C^{(3)}_\mu\,\partial_\nu C^{(4)}_\nu \\ + D^{(1)}\Box D^{(2)} + D^{(3)}\Box D^{(4)} + D^{(5)}\Box D^{(6)} + D^{(7)}\Box D^{(8)}. \qquad (3)\end{aligned}$$

This time three generations of ghosts appear; not only do the (antisymmetric) Faddeev–Popov ghosts $B^{(1)}_{\mu\nu}$ and $B^{(2)}_{\mu\nu}$ have commuting secondary ghosts $C^{(1)-(4)}_\mu$ associated with them, but also the secondary ghosts require gauge-fixing and hence anticommuting 'tertiary' ghosts $D^{(1)-(8)}$ are needed. The counting of degrees of freedom (in $3+1$ dimensions) is as follows:

$$A_{\mu\nu\lambda}: +4, \qquad B_{\mu\nu}: -12, \qquad C_\mu: +16, \qquad D: -8.$$

Summing the modes, one obtains a total of zero, in accordance with the fact that $A_{\mu\nu\lambda}$ is non-propagating. Unlike the previous example, Ward identities need not be invoked to obtain the correct count. It should be pointed out, however, that this feature is peculiar to four dimensions. In d dimensions one finds $d-4$ extra unwanted modes, which presumably decouple.

It is clear that these arguments may be generalised to an antisymmetric rank-r tensor field (in sufficiently high dimensions to prevent the Lagrangian vanishing identically, i.e. at least $r+1$). There will be r generations of ghosts, with the nth generation containing 2^n independent ghost fields which anticommute (commute) if n is odd (even). It would be of interest to extend the unitarity arguments of Townsend (1979) to cover this general case.

One situation in which these new ghost fields would become of practical importance is in background field quantum gravity (or indeed supergravity) calculations. Just as in the background field Maxwell–Einstein theory (Deser and van Nieuwenhuizen 1974), in which the normally decoupled Faddeev–Popov ghosts of Lorentz gauge electromagnetism must be included in loop diagrams, so here too every generation of ghosts will contribute non-trivially to the background functional. An interesting aspect of this would be to compare the one-loop calculations for the $A_{\mu\nu}$ field coupled to gravity with those already performed for the more conventional scalar theories (t' Hooft and Veltman 1974), and in particular to see if the issue of non-renormalisability is affected.

As a final example of a different kind, we consider a modified linear spin-2 theory, with Lagrangian

$$\mathscr{L} = \tfrac{1}{2}h_{\mu\nu,\lambda}h_{\mu\nu,\lambda} - h_{\mu\nu,\mu}h_{\lambda\nu,\lambda} + (k_1+1)h_{\mu\nu,\mu}h_{\lambda\lambda,\nu} + (k_2-\tfrac{1}{2})h_{\mu\mu,\lambda}h_{\nu\nu,\lambda} \qquad (4)$$

invariant under

$$h_{\mu\nu} \to h_{\mu\nu} + \xi_{(\mu,\nu)} \tag{5}$$

with the differential constraint

$$\partial_\mu \xi_\mu = 0 \tag{6}$$

imposed. If the arbitrary constants k_1 and k_2 are both set to zero, $\mathscr{L}$ becomes the linearised Einstein–Hilbert action, for which equation (6) is no longer required. In fact (6) has the interpretation of a volume-preserving restriction on the transformation (5), and is quite analogous to the supervolume-preserving constraint imposed in superfield supergravity, as formulated by Ogievetsky and Sokatchev (1979). Hence this 'toy' model, although containing metric ghosts (van Nieuwenhuizen 1973), is of pedagogical value in exhibiting the salient features of the role of a differential constraint in the quantisation of linearised superfield supergravity, while avoiding the somewhat involved supersymmetry formalism.

A suitable gauge-fixing constraint is

$$\partial_\mu h_{\mu\nu} - (\partial_\nu\, \partial_\lambda\, \partial_\rho/\Box) h_{\lambda\rho} = 0,$$

leading to the ghost Lagrangian

$$\mathscr{L}_{\mathrm{G}} = l'_\mu (\Box \eta_{\mu\nu} - \partial_\mu\, \partial_\nu) l_\nu.$$

The ghost gauge-fixing

$$\partial_\mu l_\mu = \partial_\mu l'_\mu = 0$$

leads in turn to the secondary ghost Lagrangian

$$\mathscr{L}'_{\mathrm{G}} = m \Box n + m' \Box n'$$

(l_μ and l'_μ are anticommuting; m, n, m', n' are commuting fields). At present it is not clear to us what an appropriate non-linear extension of (4) would be.

We end by remarking that, in the light of recent work on the geometrical interpretation of the Faddeev–Popov determinant (Babelon and Viallet 1979, Thierry-Mieg 1979), it would be interesting to understand the geometrical role of these higher-generation ghosts.

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